Ephraim and Malah suppression rule [3]

This involves deriving the MMSE STSA estimator using a complex Gaussian model of the *a priori* probability distribution of speech and noise Fourier expansion coefficients. If $y[n] = x[n] + b[n]$ and $X(k) = A_k exp(j\alpha_k)$, then the MMSE estimator of A_k is

$$\begin{aligned} \hat{A}_k &= \mathcal{E}[A_k|Y_k] \\ &= \frac{\int_0^\infty \int_0^{2\pi} a_k P(Y_k|a_k,\alpha_k) P(a_k,\alpha_k) d\alpha_k da_k}{\int_0^\infty \int_0^{2\pi} P(Y_k|a_k,\alpha_k) P(a_k,\alpha_k) d\alpha_k da_k} \end{aligned}$$

With the assumption of Fourier coefficients having a Gaussian distribution, the polar form of the coefficients have the following marginal distribution

$$P(a_k) = \begin{cases} \frac{2a_k}{\lambda_x(k)} exp(-\frac{a_k^2}{\lambda_x(k)}) & \text{if } a_k \in [0,\infty) \\ 0 & \text{otherwise} \end{cases}$$

and

$$p(\alpha_k) = \begin{cases} \frac{1}{2\pi} & \text{if } \alpha_k \in [-\pi,\pi) \\ 0 & \text{otherwise} \end{cases}$$

The prior pdf is

$$P(Y_k|a_k,\alpha_k) = \frac{1}{\pi\lambda_b(k)} exp\left\{-\frac{1}{\lambda_b(k)}|Y_k - a_k e^{j\alpha_k}|^2\right\}$$

The joint pdf is

$$P(a_k,\alpha_k) = \frac{a_k}{\pi\lambda_x(k)} exp(-\frac{a_k^2}{\lambda_x(k)})$$

The posterior density can be worked out to be

$$P(a_k|Y_k) = \frac{a_k}{\sigma_k^2} exp\left(-\frac{a_k^2 + s_k^2}{2\sigma_k^2}\right) I_0\left(\frac{a_k s_k}{\sigma_k^2}\right)$$

where

$$
\begin{aligned}
\frac{1}{\lambda(k)} &= \frac{1}{\lambda_x(k)} + \frac{1}{\lambda_b(k)} \\
\epsilon_k &= \frac{\lambda_x(k)}{\lambda_b(k)} \\
\gamma_k &= \frac{A_k^2}{\lambda_b(k)} \\
\upsilon_k &= \frac{\epsilon_k}{1+\epsilon_k}\gamma_k
\end{aligned}
$$

The authors use the first moment of the posterior distribution giving

$$\hat{A}_k = \lambda(k)^{1/2}\Gamma(1.5)\Phi(-0.5, 1; -\upsilon_k)$$

They also extend the amplitude estimator under signal presence uncertainty (see for example Maximum Likelihood estimator) but this is beyond the scope of this summary.

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